12 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1

Exploring Solids and Their Properties

Euler's Theorem is useful when finding the number of faces, edges, or vertices on a polyhedron, especially when one of those quantities is difficult to count by hand.

For example, suppose you want to find the number of edges on a regular icosahedron, which has 20 faces. You count 12 vertices on the solid. To calculate the number of edges, use Euler's Theorem:

$F + V = E + 2$ **Write Euler's Theorem.**

$20 + 12 = E + 2$ **Substitute known values.**

$30 = E$ **Solve for *E*.**

Big Idea 2

Solving Problems Using Surface Area and Volume

Figure	Surface Area	Volume
Right prism	$S = 2B + Ph$	$V = Bh$
Right cylinder	$S = 2B + Ch$	$V = Bh$
Regular pyramid	$S = B + \frac{1}{2}P\ell$	$V = \frac{1}{3}Bh$
Right cone	$S = B + \frac{1}{2}C\ell$	$V = \frac{1}{3}Bh$
Sphere	$S = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

The volume formulas for prisms, cylinders, pyramids, and cones can be used for oblique solids.

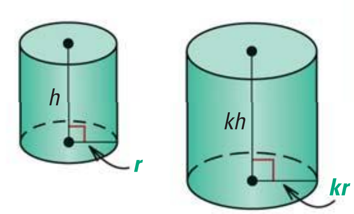
While many of the above formulas can be written in terms of more detailed variables, it is more important to remember the more general formulas for a greater understanding of why they are true.

Big Idea 3

Connecting Similarity to Solids

The similarity concepts learned in Chapter 6 can be extended to 3-dimensional figures as well.

Suppose you have a right cylindrical can whose surface area and volume are known. You are then given a new can whose linear dimensions are k times the dimensions of the original can. If the surface area of the original can is S and the volume of the original can is V, then the surface area and volume of the new can can be expressed as k^2S and k^3V, respectively.



12 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- polyhedron, *p. 794*
  face, edge, vertex, base
- regular polyhedron, *p. 796*
- convex polyhedron, *p. 796*
- Platonic solids, *p. 796*
- tetrahedron, *p. 796*
- cube, *p. 796*
- octahedron, *p. 796*
- dodecahedron, *p. 796*
- icosahedron, *p. 796*
- cross section, *p. 797*
- prism, *p. 803*
  lateral faces, lateral edges
- surface area, *p. 803*
- lateral area, *p. 803*
- net, *p. 803*
- right prism, *p. 804*
- oblique prism, *p. 804*
- cylinder, *p. 805*
- right cylinder, *p. 805*
- pyramid, *p. 810*
- vertex of a pyramid, *p. 810*
- regular pyramid, *p. 810*
- slant height, *p. 810*
- cone, *p. 812*
- vertex of a cone, *p. 812*
- right cone, *p. 812*
- lateral surface, *p. 812*
- volume, *p. 819*
- sphere, *p. 838*
  center, radius, chord, diameter
- great circle, *p. 839*
- hemisphere, *p. 839*
- similar solids, *p. 847*

VOCABULARY EXERCISES

1. Copy and complete: A __?__ is the set of all points in space equidistant from a given point.

2. **WRITING** Sketch a right rectangular prism and an oblique rectangular prism. *Compare* the prisms.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 12.

12.1 Explore Solids *pp. 794–801*

EXAMPLE

A polyhedron has 16 vertices and 24 edges. How many faces does the polyhedron have?

$F + V = E + 2$ **Euler's Theorem**

$F + 16 = 24 + 2$ **Substitute known values.**

$F = 10$ **Solve for *F*.**

▸ The polyhedron has 10 faces.

EXERCISES

EXAMPLES 2 and 3 on pp. 796–797 for Exs. 3–5

Use Euler's Theorem to find the value of *n*.

3. Faces: 20
Vertices: n
Edges: 30

4. Faces: n
Vertices: 6
Edges: 12

5. Faces: 14
Vertices: 24
Edges: n

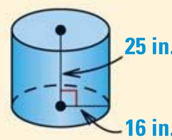
12 CHAPTER REVIEW

12.2 Surface Area of Prisms and Cylinders

pp. 803–809

EXAMPLE

Find the surface area of the right cylinder.



$S = 2\pi r^2 + 2\pi rh$	**Write formula.**
$= 2\pi(16)^2 + 2\pi(16)(25)$	**Substitute for *r* and *h*.**
$= 1312\pi$	**Simplify.**
≈ 4121.77	**Use a calculator.**

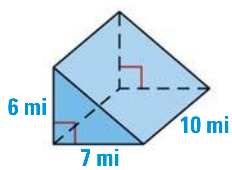
▶ The surface area of the cylinder is about 4121.77 square inches.

EXERCISES

EXAMPLES 2, 3, and 4 on pp. 804–806 for Exs. 6–9

Find the surface area of the right prism or right cylinder. Round your answer to two decimal places, if necessary.

6.



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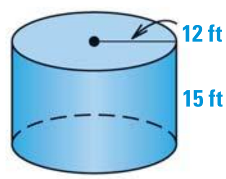

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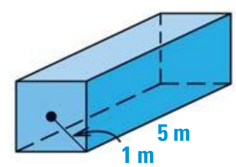

9. A cylinder has a surface area of 44π square meters and a radius of 2 meters. Find the height of the cylinder.

12.3 Surface Area of Pyramids and Cones

pp. 810–817

EXAMPLE

Find the lateral area of the right cone.

Lateral area $= \pi r\ell$	**Write formula.**
$= \pi(6)(16)$	**Substitute for *r* and ℓ.**
$= 96\pi$	**Simplify.**
≈ 301.59	**Use a calculator.**

▶ The lateral area of the cone is about 301.59 square centimeters.

EXERCISES

EXAMPLES 1, 2, and 4 on pp. 810–813 for Exs. 10–12

10. Find the surface area of a right square pyramid with base edge length 2 feet and height 5 feet.

11. The surface area of a cone with height 15 centimeters is 500π square centimeters. Find the radius of the base of the cone. Round your answer to two decimal places.

12. Find the surface area of a right octagonal pyramid with height 2.5 yards, and its base has apothem length 1.5 yards.

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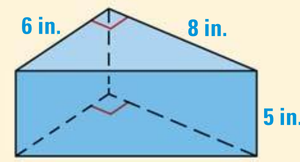
Chapter Review Practice

12.4 Volume of Prisms and Cylinders

pp. 819–825

EXAMPLE

Find the volume of the right triangular prism.



The area of the base is $B = \frac{1}{2}(6)(8) = 24$ square inches. Use $h = 5$ to find the volume.

$V = Bh$ — Write formula.

$= 24(5)$ — Substitute for B and h.

$= 120$ — Simplify.

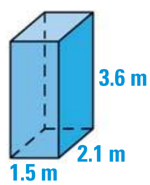
▶ The volume of the prism is 120 cubic inches.

EXERCISES

EXAMPLES 2 and 4 on pp. 820–821 for Exs. 13–15

Find the volume of the right prism or oblique cylinder. Round your answer to two decimal places.

13.



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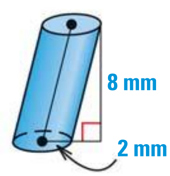

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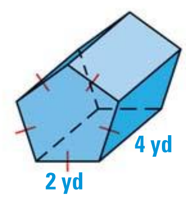
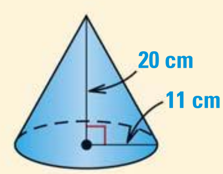


12.5 Volume of Pyramids and Cones

pp. 829–836

EXAMPLE

Find the volume of the right cone.



The area of the base is $B = \pi r^2 = \pi(11)^2 \approx 380.13\text{ cm}^2$. Use $h = 20$ to find the volume.

$V = \frac{1}{3}Bh$ — Write formula.

$\approx \frac{1}{3}(380.13)(20)$ — Substitute for B and h.

≈ 2534.20 — Simplify.

▶ The volume of the cone is about 2534.20 cubic centimeters.

EXERCISES

EXAMPLES 1 and 2 on pp. 829–830 for Exs. 16–17

16. A cone with diameter 16 centimeters has height 15 centimeters. Find the volume of the cone. Round your answer to two decimal places.

17. The volume of a pyramid is 60 cubic inches and the height is 15 inches. Find the area of the base.

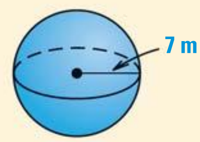
12 CHAPTER REVIEW

12.6 Surface Area and Volume of Spheres

pp. 838–845

EXAMPLE

Find the surface area of the sphere.



$S = 4\pi r^2$ — Write formula.

$= 4\pi(7)^2$ — Substitute 7 for *r*.

$= 196\pi$ — Simplify.

▶ The surface area of the sphere is 196π, or about 615.75 square meters.

EXERCISES

EXAMPLES 1, 4, and 5 on pp. 839, 841 for Exs. 18–19

18. ASTRONOMY The shape of Pluto can be approximated as a sphere of diameter 2390 kilometers. Find the surface area and volume of Pluto. Round your answer to two decimal places.

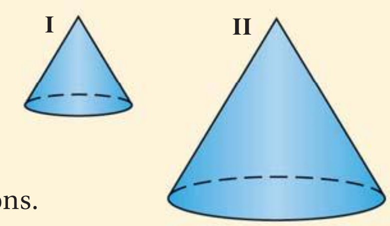
19. A solid is composed of a cube with side length 6 meters and a hemisphere with diameter 6 meters. Find the volume of the solid. Round your answer to two decimal places.

12.7 Explore Similar Solids

pp. 847–854

EXAMPLE

The cones are similar with a scale factor of 1 : 2. Find the surface area and volume of Cone II given that the surface area of Cone I is 384π square inches and the volume of Cone I is 768π cubic inches.



Use Theorem 12.13 to write and solve two proportions.

$$\frac{\text{Surface area of I}}{\text{Surface area of II}} = \frac{a^2}{b^2} \qquad \frac{\text{Volume of I}}{\text{Volume of II}} = \frac{a^3}{b^3}$$

$$\frac{384\pi}{\text{Surface area of II}} = \frac{1^2}{2^2} \qquad \frac{768\pi}{\text{Volume of II}} = \frac{1^3}{2^3}$$

$$\text{Surface area of II} = 1536\pi \text{ in.}^2 \qquad \text{Volume of II} = 6144\pi \text{ in.}^3$$

▶ The surface area of Cone II is 1536π, or about 4825.48 square inches, and the volume of Cone II is 6144π, or about 19,301.93 cubic inches.

EXERCISES

EXAMPLE 2 on p. 848 for Exs. 20–22

Solid A is similar to Solid B with the given scale factor of A to B. The surface area and volume of Solid A are given. Find the surface area and volume of Solid B.

20. Scale factor of 1 : 4
$S = 62 \text{ cm}^2$
$V = 30 \text{ cm}^3$

21. Scale factor of 1 : 3
$S = 112\pi \text{ m}^2$
$V = 160\pi \text{ m}^3$

22. Scale factor of 2 : 5
$S = 144\pi \text{ yd}^2$
$V = 288\pi \text{ yd}^3$